Conservation-Congruent Encodings

Peter David Fagan, School of Informatics, University of Edinburgh

April 24, 2026

Abstract

A conservation-congruent encoding (CCE) treats information as a physically realized macroscopic distinction rather than a substrate-independent abstraction. Under a chosen coarse-graining, a CCE consists of protected macroscopic regions and world-tubes that survive ambient fluctuations, are stabilized by invariants tied to conserved quantities, and exchange conserved loads with modeled channels when macroscopic support is compressed or expanded. Shannon entropy still measures logical distinguishability, but only as the symmetric limit of a broader physical ledger whose nat count is defined relative to the substrate's equilibrium reference state. This note gives a minimal definition of a CCE.

1 Introduction

Classical information theory treats information as substrate-independent: Shannon entropy measures logical uncertainty, not the physical strain stored in a substrate. Yet information is always realized as a distinction carried by matter. Landauer's analysis of logical irreversibility and Bennett's reversible-computation perspective show that persistence, reversible transport, and erasure have different physical statuses [13, 3]. The question, then, is how to describe information processing directly in matter. The Conservation-Congruent Encoding (CCE) framework developed here, and in a companion preprint [7], answers that question by defining information through coarse-grained distinctions that persist against fluctuations and can be irreversibly merged only at measurable physical cost.

This does not discard Shannon bookkeeping; it locates it. Shannon entropy still measures loss of logical distinguishability under a reset map, but by itself it is blind to equilibrium asymmetry, stored nonequilibrium strain, and channel-specific exchange. Those quantities appear only relative to the equilibrium reference state selected by the substrate and its reservoirs. The abstract account is therefore recovered as a symmetric limit of a broader physical ledger rather than taken as the default description of information in matter.

That broader ledger is not new. In modern stochastic thermodynamics, the identification of $k_B T\, D_{\mathrm{KL}}(\rho \,\|\, \rho^{\mathrm{eq}})$ with a nonequilibrium free-energy offset and generalized Landauer bookkeeping is already well established [19, 6, 18]. The novelty here is to package that established ledger into a geometric world-tube criterion, together with a metriplectic split, that states when a macroscopic distinction counts as a physical encoding and how reversible transport differs from irreversible merger.

A macroscopic distinction must first persist. Once it does, the dynamics may either transport it reversibly within a protected region, merge it irreversibly across a boundary, or physically expand support for a retrodictive one-to-many operation in a way consistent with the substrate's conservation structure. Metriplectic dynamics capture that split [16, 17]: the Poisson sector transports a protected state without destroying its identity, while the metric sector is the thermodynamic port through which merger exports conserved load and expansion imports it. The rest of this note states the minimal conditions under which a macroscopic distinction qualifies as a conservation-congruent encoding.

2 Metriplectic Dynamics of Information

Let X denote the microscopic dynamical state space of the substrate, with microscopic state $x(t) \in X$ evolving under the fine-grained flow

$$\dot{x} = F(x). \tag{1}$$

Let the chosen mesoscopic coarse-graining define a readout space $\mathcal{Z}$ of slow, operationally resolvable variables. The readout projection is

$$\pi : X \to \mathcal{Z}, \qquad Z = \pi(x). \tag{2}$$

The logical CCE sits one tier higher. A protected partition of $\mathcal{Z}$ induces a discrete logical state space $\Sigma = \{0, \ldots, m-1\}$ and a logical projection

$$\Pi_t : X \to \Sigma, \qquad \Pi_t = \chi_t \circ \pi, \tag{3}$$

where $\chi_t(Z) = i$ exactly when $Z \in \mathcal{M}_i(t)$. Thus X is microscopic, $\mathcal{Z}$ is mesoscopic, and $\Sigma_t = \Pi_t(x) \in \Sigma$ is the macroscopic logical state. Integrating the mesoscopic density over $\mathcal{M}_i(t)$ gives $P_i(t) = \Pr[\Sigma_t = i]$, and $\mathcal{W}_i = \bigcup_t (\mathcal{M}_i(t) \times \{t\})$ is the corresponding world-tube. Throughout, $\rho(Z,t)$ and $\rho^{\mathrm{eq}}(Z)$ denote densities on $\mathcal{Z}$, whereas $P(t)$ and P^{eq} denote the induced probabilities on the logical classes of Σ. If several conserved-quantity channels are relevant, one may multiplex the readout as $\pi = (\pi^{(1)}, \ldots, \pi^{(C)})$ with $\mathcal{Z} = \mathcal{Z}^{(1)} \times \cdots \times \mathcal{Z}^{(C)}$. It is important to keep Z, the logical state Σ_t, and any channel-ledger variables distinct: the first are readout coordinates, the second label logical classes, and the third record conserved quantities exchanged with modeled channels along a protocol. A CCE is therefore defined on the hierarchical coarse-graining (π, Π_t) together with the protected regions that realize it. The fast degrees of freedom discarded by π remain as the unresolved bath.

For any smooth mesoscopic observable $A(Z)$, the effective dynamics of the retained observables are written in metriplectic form,

$$\dot{A} = \{A, H\} + (A, S), \tag{4}$$

and, at the level of a deterministic drift skeleton or expectation dynamics, for the mesoscopic coordinates,

$$\dot{Z}^a = \{Z^a, H\} + (Z^a, S), \tag{5}$$

or compactly $\dot{Z} = \{Z, H\} + (Z, S)$. This trajectory form is only a skeleton. When ambient noise is operationally relevant, the state is a probability density $\rho(Z,t)$ on $\mathcal{Z}$ and the metriplectic structure must be lifted from observables $A(Z)$ to functionals $A[\rho]$. Section 4 makes that lift explicit for the Smoluchowski colloidal bit, where Kramers escape and KL descent live on the density manifold rather than on a single noiseless coordinate. Here H generates the reversible sector and S is an entropy-like potential that orders irreversible relaxation. The Poisson term describes reduced nondissipative transport on the retained mesoscopic manifold, whereas the metric term is the open-system contact with the eliminated degrees of freedom.

For CCE bookkeeping, the metric bracket should therefore be read as a bidirectional thermodynamic port linking the observer-controlled macroscopic variables to the unresolved bath. It is not merely an exhaust pipe. When the observer executes a many-to-one compression protocol, the port exports the conserved load required to make the merger irreversible. When the observer executes a one-to-many expansion or retrodictive protocol, the same port carries the conjugate load back into the retained support, with the sign of the ledger reversed in the quasistatic inverse.

The reversible sector is encoded by the Poisson bracket

$$\{A, B\} = \partial_a A \, J^{ab}(Z) \, \partial_b B, \qquad J^{ab} = -J^{ba}, \tag{6}$$

with J the antisymmetric Poisson tensor. The irreversible sector is encoded by the symmetric bracket

$$(A, B) = \partial_a A \, M^{ab}(Z) \, \partial_b B, \qquad M^{ab} = M^{ba}, \quad M \succeq 0, \tag{7}$$

with M the positive semi-definite metric operator. In this form the symmetric bracket is the coarse-grained bookkeeping device for friction, relaxation, and bidirectional conserved-load exchange with the environment.

For the closed reduced description, or equivalently for the isolated system-plus-reservoir super-system before projection, the standard metriplectic degeneracy conditions are

$$\{S, A\} = 0, \qquad (H, A) = 0, \tag{8}$$

for admissible observables A. These are independent axioms. The first makes S a Casimir invariant of the Poisson sector, so the reversible flow is isentropic; the second makes the dissipative sector preserve the conserved generator. Together they imply

$$\dot{H} = 0, \qquad \dot{S} = (S, S) \geq 0. \tag{9}$$

For an open subsystem, however, one does not impose $(H, A) = 0$ on the subsystem energy itself. Heat or other conserved quantities can flow through the eliminated channels, so a local system energy H_{sys} or $U(Z)$ generally satisfies $\dot{H}_{\mathrm{sys}} \neq 0$. The reduced open-system statement is instead that the metric sector dissipates a free-energy-like functional at fixed reservoir parameters, or equivalently that $(H_{\mathrm{tot}}, A) = 0$ holds only on the enlarged isolated description while the subsystem obeys a balance law with exchange terms [16, 17, 9, 19, 6]. Throughout, S is the entropy-like quantity raised by the metric sector. If one instead writes a free-energy-like descent functional $\Xi \equiv -S$, the same positive symmetric bracket appears as $\dot{\Xi} = -(\Xi, \Xi) \leq 0$. The Smoluchowski specialization later uses exactly this open-system convention with $S_\lambda[\rho] = -D_{\mathrm{KL}}(\rho \parallel \rho_\lambda^{\mathrm{eq}})$, so monotone KL decay is the concrete realization of $\dot{S} = (S, S) \geq 0$ at fixed reservoir parameters rather than a statement that the bead's local energy is conserved.

This is the minimal dynamical split needed for information. The Poisson sector preserves the invariants that stabilize an encoding, whereas the metric sector governs dissipative coarsening and the thermodynamic port load when control protocols change macroscopic support. During many-to-one merger, phase-space volume lost from the retained description is not destroyed; it is exported into environmental degrees of freedom excluded by π. Hence $\dot{S} = (S, S) \geq 0$ is a coarse-grained signature of dissipative exchange compatible with Landauer-type lower bounds once a reservoir model, control protocol, and reset map are specified [13, 22]. In a thermal reservoir this load appears as heat, in a spin reservoir as angular momentum, and in other substrates as the relevant conserved quantity exchanged against its conjugate generalized force.

The inverse operation must not be confused with algebraically replacing t by $-t$ in the reduced equations. Such a sign flip would turn the positive metric sector into negative mobility—diffusion into anti-diffusion—and would therefore require a magic instability rather than a physical bath. True retrodiction is instead implemented by a time-reversed control protocol $\lambda^\dagger(t)$ that actively lowers barriers, opens phase-space support, and lets the state diffuse under the same positive metric bracket. The Second Law statement $\dot{S} \geq 0$ is not suspended; what changes is the sign of the exchanged load at the port. In an isothermal expansion, the metric sector acts as an intake valve: thermal energy is imported from the bath to sustain the enlarged support while the controller maintains the reservoir parameters.

When several channels are active simultaneously, the same structure applies on the multiplexed space $\mathcal{Z} = \mathcal{Z}^{(1)} \times \cdots \times \mathcal{Z}^{(C)}$. The metric operator may be block-diagonal when channels erase independently or mixed when one logical distinction is stabilized by coupled dynamics. At the density level the corresponding equilibrium reference state is a generalized Gibbs ensemble, $\rho^{\mathrm{eq}}_{\boldsymbol{\lambda}}(Z) \propto \exp[-\sum_c \lambda_c X_c(Z)]$, where each λ_c is the Gibbs multiplier conjugate to X_c fixed by the chosen reservoir model. The exact physical nat count is then $D_{\mathrm{KL}}(\rho \parallel \rho^{\mathrm{eq}}_{\boldsymbol{\lambda}})$. After coarse-graining into logical classes, the induced equilibrium weights inherit the same multipliers together with basin-degeneracy factors. Channel-resolved exports or imports are then read from the conjugate pairs (X_c, λ_c) once the reservoir convention is specified.

3 The Geometric Conditions for a Conservation-Congruent Encoding

Under a chosen logical coarse-graining $\Pi_t : X \to \Sigma$, realized by protected regions $\{\mathcal{M}_i(t)\} \subset \mathcal{Z}$ and associated world-tubes $\{\mathcal{W}_i\}$, a Conservation-Congruent Encoding exists if and only if three defining conditions hold. They provide a compact roadmap from encoding to reversible manipulation to irreversible erasure and controlled retrodictive expansion:

Condition I: Metastable Confinement

Protected mesoscopic cross-sections, each realizing a macroscopic logical class, must assemble into a world-tube that survives throughout the operational window with exponentially suppressed escape.

Condition II: Transverse Restoring Structure

Each protected region must admit local charts whose transverse restoring geometry makes quasistatic deformations well posed and drives the excess channel expenditure to zero in the quasistatic limit.

Condition III: Irreversible Tube Merger and Expansion

A logically irreversible operation is physically realized only when two or more disjoint world-tubes are collapsed into a common output tube, with the metric sector exporting the conserved load that makes the merger irreversible. The physical inverse is not algebraic time reversal but a controlled one-to-many expansion that imports the corresponding load through the same metric port.

Individual readout states $Z \in \mathcal{M}_i(t)$ are representatives of the encoding, not the encoding itself. To make the temporal aspect explicit, define the extended mesoscopic space

$$\mathcal{E} = \mathcal{Z} \times \mathcal{T}, \tag{10}$$

with coordinates (Z, t). In this view, a logical state is not merely a basin in $\mathcal{Z}$ at an instant, but a protected world-tube

$$\mathcal{W}_i = \bigcup_{t \in [t_0, t_0 + \tau_{\mathrm{op}}]} (\mathcal{M}_i(t) \times \{t\}) \subset \mathcal{E}, \tag{11}$$

whose lateral boundary is the time-extended separatrix $\bigcup_{t\in[t_0,t_0+\tau_{\rm op}]}(\partial\mathcal{M}_i(t)\times\{t\})$. The stationary case is recovered by $\mathcal{M}_i(t)\equiv\mathcal{M}_i$. A CCE persists when the trajectory reaches the terminal slice $t_0+\tau_{\rm op}$ without crossing that boundary.

3.1 Condition I: Metastable Confinement

Condition I is the encoding criterion. A valid encoding is a protected family of macroscopic cross-sections $\mathcal{M}_i(t)\subset\mathcal{Z}$ whose union along $\mathcal{T}$ forms a metastable world-tube $\mathcal{W}_i\subset\mathcal{E}$. For a sample path $t\mapsto Z(t)$ of the reduced stochastic dynamics, define the operational escape probability by

$$\begin{aligned}
P^{(i)}_{\rm esc}(\tau_{\rm op}) &\equiv \Pr\left(\exists\, t\in[t_0,t_0+\tau_{\rm op}] : (Z(t),t)\notin\mathcal{W}_i \mid Z(t_0)\in\mathcal{M}_i(t_0)\right),\\
P^{(i)}_{\rm esc}(\tau_{\rm op}) &\lesssim \tau_{\rm op}\,\nu_{0,i}\exp\left(-\frac{\Delta\mathcal{B}_i}{\sigma_i}\right), \qquad \tau_{\rm op}\ll\tau_{\rm escape}.
\end{aligned} \tag{12}$$

Here $\nu_{0,i}$ is the intrawell attempt frequency, $\Delta\mathcal{B}_i$ the channel-resolved quasi-potential barrier along the dominant escape path, and σ_i the corresponding fluctuation scale. In a weak-noise reduction with Kramers-type escape one typically has $\tau_{\rm escape}^{-1}\sim\nu_{0,i}\exp(-\Delta\mathcal{B}_i/\sigma_i)$, so Eq. (12) should be read as a model-dependent short-window first-passage estimate for approximately Poissonian escape, not as a universal consequence of coarse-graining. The Arrhenius–Kramers thermal limit is recovered by $\Delta\mathcal{B}_i=\Delta E_i$ and $\sigma_i=k_BT$. When the eliminated fast variables admit a near-equilibrium fluctuation-dissipation closure, M^{ab} acts as a mobility tensor and one has an Einstein relation of the form $D^{ab}=k_BT\,M^{ab}$ (up to conventions for S), so the same mobility/diffusion data govern both escape and metric dissipation. Outside that regime the relation is model dependent.

The tube length along the time axis defines the operational window. Inside $\mathcal{W}_i$, the state may drift or circulate without loss of logical identity; what matters is confinement until the terminal slice. Writing $\tau_{\rm escape}$ for the mean first-passage time across the tube boundary and $\tau_{\rm relax}$ for the local relaxation time of the fast variables eliminated by π, the defining timescale separation is

$$\tau_{\rm relax}\ll\tau_{\rm op}\ll\tau_{\rm escape}. \tag{13}$$

3.2 Condition II: Transverse Restoring Structure

Condition II is the reversible-manipulation criterion. Along the operating portion of each world-tube, there must be an atlas of local charts in which the protected section is transversely restoring and can be advected without loss of logical identity. Concretely, for each reference point $Z_*(t)\in\mathcal{M}_i(t)$ there must exist a chart

$$\psi_k: U_k\to\mathbb{R}^{d_t}\times\mathbb{R}^{d_n}, \qquad u=\psi_k(Z)=(s,r), \tag{14}$$

where s parameterizes directions tangent to the intended transport and r parameterizes directions normal to the tube, together with an effective quasi-potential $\Phi_k(u;\lambda)$. This Φ_k is not an extra global generator beside H and S; it is the local transverse representative of the same dissipation geometry introduced in Section 2. Equivalently, after freezing λ and restricting to the chart U_k, one may regard Φ_k as the local restriction of the free-energy-like descent functional $\Xi=-S$, up to additive constants and tangential terms that do not affect the normal restoring force. In thermal Smoluchowski reductions it becomes the local free-energy or potential landscape that also controls escape. Here ∇_r and ∇^2_{rr} denote the gradient and Hessian with respect to the transverse coordinates r, evaluated at the tube center $r=0$, so the local transverse conditions are

$$\nabla_r\Phi_k(s,0;\lambda)=0, \qquad \nabla^2_{rr}\Phi_k(s,0;\lambda)\succeq\kappa_k I, \qquad \kappa_k>0. \tag{15}$$

This is a local transverse-convexity requirement. Along tangent directions the tube may translate, shear, or slowly reshape under slow deformation, but in the normal directions there must remain restoring structure that keeps nearby trajectories slaved to the protected section. Global non-convexity is therefore not excluded; indeed, it is what allows distinct logical world-tubes to coexist.

Let $\lambda(t)$ be a deformation schedule that moves the reference section through this atlas without destroying the positivity of the transverse Hessian. If the schedule is written as a fixed-shape family $\lambda_{\tau_{\rm op}}(t) = \lambda(t/\tau_{\rm op})$, the quasistatic limit is obtained by stretching the duration rather than changing the path. In that formal limit,

$$\sup_{t\in[0,\tau_{\rm op}]} \left\| \dot{\lambda}_{\tau_{\rm op}}(t) \right\| = O(\tau_{\rm op}^{-1}), \qquad \tau_{\rm relax} \ll \tau_{\rm op}, \tag{16}$$

so the state tracks the moving protected section and the excess conserved-quantity expenditure above the reversible transport limit vanishes:

$$C_{\rm ex}[\lambda_{\tau_{\rm op}}] \to 0 \qquad \text{as} \qquad \tau_{\rm op} \to \infty. \tag{17}$$

This reversibility statement must be read together with Condition I, not as a replacement for it. In a fluctuating thermal memory one still requires $\tau_{\rm op} \ll \tau_{\rm escape}$ to keep the logical world-tube intact. At fixed barrier height and noise strength, taking $\tau_{\rm op}$ arbitrarily large eventually gives the system enough time to escape spontaneously, so the same bath that permits quasistatic relaxation also erodes the stored distinction. The literal limit $C_{\rm ex} \to 0$ is therefore unavailable for a fixed finite-barrier device unless protection is strengthened simultaneously so that $\tau_{\rm escape}$ grows with $\tau_{\rm op}$ (or the noise level is sent to zero). Operational computing thus lives in the finite window $\tau_{\rm relax} \ll \tau_{\rm op} \ll \tau_{\rm escape}$ and consequently incurs a finite excess cost. When linear-response assumptions apply, the excess expenditure has the standard quadratic form

$$C_{\rm ex}[\lambda] \approx \int_0^{\tau_{\rm op}} \dot{\lambda}^{\alpha}(t)\, \zeta_{\alpha\beta}(\lambda_t)\, \dot{\lambda}^{\beta}(t)\, {\rm d}t, \tag{18}$$

with $\zeta_{\alpha\beta}$ a positive semi-definite thermodynamic-friction tensor [20]. Any finite control speed $\dot{\lambda} \neq 0$ therefore produces a nonzero lag between the actual state and the instantaneous equilibrium state selected by λ_t: the control moves the protected section, and the metric sector must continuously relax the displaced distribution back toward it. For a fixed path this yields the thermodynamic-length lower bound

$$C_{\rm ex}[\lambda] \geq \frac{\mathcal{L}[\lambda]^2}{\tau_{\rm op}}, \qquad \mathcal{L}[\lambda] = \int_0^{\tau_{\rm op}} \sqrt{\dot{\lambda}^{\alpha} \zeta_{\alpha\beta}(\lambda_t) \dot{\lambda}^{\beta}}\, {\rm d}t, \tag{19}$$

so $C_{\rm ex} = 0$ is attained only in the infinite-time limit. The familiar scaling $C_{\rm ex} = O(\tau_{\rm relax}/\tau_{\rm op})$ is therefore not a loophole but the finite-time friction law itself: time is the resource that buys down the metric cost of transport. Condition II does not require the full logical landscape to be convex. It requires only that each protected tube admit a local atlas in which transverse restoring geometry prevents leakage and sufficiently slow deformation suppresses excess dissipation.

3.3 Condition III: Irreversible Tube Merger and Expansion

Condition III is the erasure criterion and, by controlled inversion, the expansion criterion. It is met in the forward direction when the basin structure in $\mathcal{E}$ is deformed so that two or more disjoint input world-tubes can no longer remain separate protected classes and instead terminate in a common output tube. In the extended-space picture this is the familiar pair-of-pants geometry: distinct legs in the past join into one trunk in the future. This intended merger is distinct from the noise-induced escape of Condition I; during a reset, crossing the former separatrix may be part of the protocol because the boundary itself has been lowered, displaced, or removed.

The logical inverse of the pair-of-pants merger is a one-to-many expansion. An observer who receives the output state and attempts to deduce which input tube it came from must physically reopen macroscopic support across the previously lowered barrier, rather than merely run the reduced equations backward. The required control protocol expands the protected support and lets probability diffuse into the admissible predecessor basins. In that retrodictive operation the metric sector is still dissipative in the mathematical sense $\dot{S} \geq 0$, but it acts as an intake port: conserved load is imported from the modeled bath to sustain the expanded support selected by the time-reversed control protocol.

On any constant-time slice, such a collapse identifies previously distinct macroscopic classes and therefore reduces coarse-grained distinguishability. If the input partition has Shannon entropy $\mathcal{H}_{\text{in}}$ and the output partition induced by this merger has entropy $\mathcal{H}_{\text{out}}$, the corresponding coarse-grained Shannon-entropy drop in nats is

$$\Delta\mathcal{H}_{\text{cg}} = \mathcal{H}_{\text{in}} - \mathcal{H}_{\text{out}}. \tag{20}$$

For the special case of a complete merger of m input classes into a single output class with prior probabilities $\{p_i\}_{i=0}^{m-1}$, this reduces to

$$\Delta\mathcal{H}_{\text{cg}} = -\sum_{i=0}^{m-1} p_i \ln p_i. \tag{21}$$

Two cases should be kept separate. If only one basin carries support on the initial slice—for example $p_0 = 1$ and $p_1 = 0$—then lowering an intervening barrier and carrying the occupied basin into the output basin is only a one-to-one transport of the realized macroscopic state. The occupied coarse-grained phase-space volume does not contract, so no logical erasure occurs even if an unoccupied basin is geometrically removed. Erasure begins only when two or more protected input classes carry weight. If $p_0, p_1 > 0$, the disjoint occupied support $\mathcal{M}_0(t_0) \cup \mathcal{M}_1(t_0)$ is compressed into a single output support, and the lost retrodictability cannot remain in the retained macroscopic description; it must be exported through the metric sector as a dissipative port load. Conversely, a retrodictive one-to-many protocol starts from a single occupied output support and expands it across the branch boundary. The missing branch variable can become physically represented only if the observer supplies the macroscopic volume and imports the conjugate conserved load through the same metric sector. In physical realizations these prior weights need not be abstract free parameters; they may be induced by a preparation procedure or by an operational quasi-stationary density on the coarse-grained landscape. If $\varrho(Z, t_0)$ denotes that density on the initial slice, then

$$p_i = \int_{\mathcal{M}_i(t_0)} \varrho(Z, t_0)\, dZ. \tag{22}$$

Basin geometry therefore enters the accounting indirectly through the induced weights, whereas barrier heights, curvatures, and protocol details enter metastability and finite-time dissipation directly. Exact bookkeeping in the canonical thermal channel instead compares the actual coarse-grained state with the equilibrium state selected by the landscape. Let

$$P(t) = (p_0(t), \ldots, p_{m-1}(t)), \qquad P_\lambda^{\text{eq}} = (p_0^{\text{eq}}(\lambda), \ldots, p_{m-1}^{\text{eq}}(\lambda)), \tag{23}$$

with equilibrium weights

$$p_i^{\mathrm{eq}}(\lambda) = \int_{\mathcal{M}_i(\lambda)} \frac{e^{-\beta U(Z;\lambda)}}{\int_{\mathcal{Z}} e^{-\beta U(Z';\lambda)}\, dZ'}\, dZ, \qquad \beta = (k_B T)^{-1}. \tag{24}$$

The physical nat count retained in the coarse-grained state is then

$$\mathcal{N}[P;\lambda] \equiv D_{\mathrm{KL}}(P \parallel P_\lambda^{\mathrm{eq}}) = \sum_{i=0}^{m-1} p_i \ln \frac{p_i}{p_i^{\mathrm{eq}}(\lambda)}. \tag{25}$$

This discrete quantity is the logical-level ledger induced by the partition. In general it is bounded above by the exact density-level nat count,

$$D_{\mathrm{KL}}(P \parallel P_\lambda^{\mathrm{eq}}) \le D_{\mathrm{KL}}(\rho \parallel \rho_\lambda^{\mathrm{eq}})\,, \tag{26}$$

with equality only when the coarse-graining is an exact lumping, for example when intra-basin nonequilibrium structure is absent or has relaxed to local equilibrium so that the occupancies P carry all of the displacement from equilibrium. For a single thermal channel, when this exact-lumping condition holds, the associated nonequilibrium free-energy offset [19, 6, 18] is

$$\mathcal{J}_{T,\min}^{\mathrm{CCE}}[P;\lambda] = k_B T\, \mathcal{N}[P;\lambda]. \tag{27}$$

For a quasistatic protocol that carries $(P_{\mathrm{in}}, \lambda_{\mathrm{in}})$ to $(P_{\mathrm{out}}, \lambda_{\mathrm{out}})$, the corresponding released thermal-channel availability is therefore, under the same exact-lumping assumption,

$$\Delta\mathcal{J}_{T,\min}^{\mathrm{CCE}} = k_B T\Big[D_{\mathrm{KL}}\Big(P_{\mathrm{in}} \parallel P_{\lambda_{\mathrm{in}}}^{\mathrm{eq}}\Big) - D_{\mathrm{KL}}\Big(P_{\mathrm{out}} \parallel P_{\lambda_{\mathrm{out}}}^{\mathrm{eq}}\Big)\Big]. \tag{28}$$

The exact ledger behind the optical-tweezer example discussed in Section 4.1 is the density-level KL functional introduced below. The discrete KL divergence here is its two-state reduction: exact when the basins form a locally equilibrated lumping, and otherwise a coarse-grained proxy or lower bound. The coarse-grained Shannon drop $\Delta\mathcal{H}_{\mathrm{cg}}$ still records the abstract nat loss of the logical merger, but it is only a symmetric proxy for the physical expenditure. In the unbiased two-well limit $P_\lambda^{\mathrm{eq}} = (1/2, 1/2)$; for a bead known to occupy one well, $D_{\mathrm{KL}}((0,1) \parallel (1/2,1/2)) = \ln 2$, so Eq. (27) recovers the usual $k_B T \ln 2$ benchmark.

When several channels are active simultaneously, the same construction survives coarse-graining: the equilibrium logical weights take generalized Gibbs form, now with an explicit factor for the internal degeneracy of each coarse-grained basin. Writing $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_C)$ for the Gibbs multipliers fixed by the chosen reservoirs, $X_c(i)$ for the coarse-grained value of conserved quantity X_c in logical class i, and Ω_i for the internal density of states of basin $\mathcal{M}_i$, one has

$$P_{\boldsymbol{\lambda}}^{\mathrm{eq}}(i) = \frac{\Omega_i\, \exp\Big[-\sum_{c=1}^{C} \lambda_c X_c(i)\Big]}{\sum_{j=0}^{m-1} \Omega_j\, \exp\Big[-\sum_{c=1}^{C} \lambda_c X_c(j)\Big]}, \qquad \mathcal{N}[P;\boldsymbol{\lambda}] = D_{\mathrm{KL}}(P \parallel P_{\boldsymbol{\lambda}}^{\mathrm{eq}})\,. \tag{29}$$

At the logical level, this KL divergence measures the retained nat count relative to the coupled reservoirs and coincides with the exact density-level count only when the coarse-graining is exact. Equivalently, one may absorb Ω_i into an effective class free energy, so the discrete formula remains consistent with the basin-volume integral in Eq. (24). The multipliers λ_c fix the dimensionless weights inside the equilibrium denominator; any factors of β and the sign convention belong to the chosen reservoir model. A specified reservoir model and control protocol are still needed to translate a change in $\mathcal{N}$ into imports or exports in heat, spin, flux, chemical work, or other channels, but they do not alter the nat count itself. In the unbiased thermal two-well limit, Eqs. (25) and (27) reduce to the usual Landauer relation with coarse-grained nat count $\ln 2$ and $Q_{\min} = k_B T \ln 2$ [13]. The label "conservation-congruent" indicates that the same conservation structure both stabilizes the encoding geometry and governs the exchanged load when world-tubes are merged or expanded.

Together, the preceding conditions yield a direct dictionary between metriplectic operators, encoding geometry, and the nat ledger. Condition II is governed by the Poisson sector, which transports a protected state along a world-tube while preserving the invariants that keep the logical class identifiable; in the ideal limit it does so without entropy production because $\{S, A\} = 0$. Conditions I and III make the metric sector operationally visible: in Condition I, the mobility and fluctuation structure in M^{ab} sets the leakage rate across the tube boundary, while in Condition III the symmetric bracket is the port through which many-to-one merger exports conserved load and one-to-many expansion imports it.

In the thermal optical-tweezer reduction discussed below, that dictionary becomes a continuous-time ledger for an open isothermal subsystem on the manifold of densities $\rho(Z, t)$ rather than for a single deterministic bead coordinate; equivalently, the metriplectic structure is lifted to a Wasserstein-type density dynamics [11]. The control sector reshapes the potential $U(Z; \lambda_t)$ and thereby moves the instantaneous equilibrium distribution selected by the landscape. The bead's local energy is therefore not conserved: the metric sector relaxes the actual density toward that moving target while exchanging heat with the bath. If $D_{\mathrm{KL}}(\rho_t \parallel \rho^{\mathrm{eq}}_{\lambda_t})$ denotes the exact physical nat content stored in the nonequilibrium density, the control protocol changes that quantity by moving the reference state; the metric flow decreases it while the sign of the heat ledger depends on whether the protocol compresses or expands the retained support.

4 Illustrative Physical Realizations

To anchor the definition in hardware, this section first studies an explicitly asymmetric single-bit realization, then uses the symmetric limit as a benchmark, and finally gives a cross-substrate bookkeeping summary. A colloidal bead in a tunable optical double-well potential provides the thermal single-channel case used in experimental Landauer tests [4, 12]. Table 1 then summarizes how the same CCE geometry pairs with different conserved quantities and generalized forces across superconducting, spintronic, electrochemical, and hybrid devices.

4.1 The Asymmetric Colloidal Bit: Physics and Accounting

This example is easiest to read in four steps: define the protected logical partition, write the continuous metriplectic ledger, follow the reset surgery, and then compare the physical and abstract accounts.

4.1.1 Readout, Basins, and Bias

The colloidal bead in a driven double-well potential is the cleanest pedagogical example of a CCE. Microscopically, X contains the bead and fluid bath. The mesoscopic readout projection π retains only the bead position Z on timescales long compared with momentum relaxation, so $\mathcal{Z} \cong \mathbb{R}$. The macroscopic logical state is the world-tube index $\Sigma_t \in \{0, 1\}$ induced by the protected partition of the potential. If $Z^\ddagger(t)$ is the instantaneous saddle point, define

$$\mathcal{M}_0(t) = \{Z < Z^\ddagger(t)\}, \qquad \mathcal{M}_1(t) = \{Z > Z^\ddagger(t)\}, \tag{30}$$

and set

$$\Sigma_t(Z) = \begin{cases} 0, & Z \in \mathcal{M}_0(t), \\ 1, & Z \in \mathcal{M}_1(t). \end{cases} \tag{31}$$

The logical projection is therefore $\Pi_t = \Sigma_t \circ \pi : X \to \{0,1\}$. A realized point $Z(t)$ tells us which basin is occupied, but the CCE itself is the pair of protected basins and the world-tubes they trace out in $\mathcal{E} = \mathcal{Z} \times \mathcal{T}$. In the regime emphasized here, the landscape is intentionally biased: basin $\mathcal{M}_0$ is deeper and passive, whereas basin $\mathcal{M}_1$ is shallower and actively prepared. Writing the minimum locations as $Z_0(\lambda)$ and $Z_1(\lambda)$, define the bias offset

$$\Delta U_{\text{bias}}(\lambda) = U(Z_1(\lambda);\lambda) - U(Z_0(\lambda);\lambda) > 0. \tag{32}$$

Occupying $\mathcal{M}_1$ therefore stores a nonzero thermal-channel offset relative to the favored basin $\mathcal{M}_0$. In the two-state coarse-grained model, and for comparable intrawell partition factors, the equilibrium occupancy ratio obeys

$$\frac{p_1^{\text{eq}}(\lambda)}{p_0^{\text{eq}}(\lambda)} \approx e^{-\beta \Delta U_{\text{bias}}(\lambda)}. \tag{33}$$

Thus a landscape with $p_0^{\text{eq}} = 0.999$ and $p_1^{\text{eq}} = 0.001$ has a coarse-grained free-energy bias $\Delta U_{\text{bias}} \approx k_B T \ln(999) \approx 6.91\, k_B T$. The symmetric textbook bit is recovered only when $\Delta U_{\text{bias}} = 0$ and the preparation is symmetric, $p_0 = p_1 = 1/2$.

4.1.2 The Continuous Metriplectic Ledger

To connect this example with the metriplectic split, let $U(q;\lambda(t))$ be the controlled optical potential and write

$$H_\lambda(q,p) = \frac{p^2}{2m} + U(q;\lambda(t)), \qquad \{A,B\}_{\text{can}} = \partial_q A\, \partial_p B - \partial_p A\, \partial_q B. \tag{34}$$

At the level of the phase-space density $f(q,p,t)$, the bead dynamics take the split form

$$\partial_t f = -\{f, H_\lambda\}_{\text{can}} + \partial_p \left[\frac{\gamma}{m} p f + \gamma k_B T\, \partial_p f \right]. \tag{35}$$

The first term is reversible Poisson transport generated by H_λ, while the second is the dissipative friction-diffusion sector that exchanges energy with the bath. After projecting to the retained coordinate $Z = q$, the coarse-grained dynamics are

$$\gamma \dot{Z} = -\partial_Z U(Z;\lambda(t)) + \sqrt{2\gamma k_B T}\, \xi(t), \tag{36}$$

with $\langle \xi(t) \rangle = 0$ and $\langle \xi(t)\xi(t') \rangle = \delta(t-t')$. Here the logical register is $\Sigma_t \in \{0,1\}$, realized by the partition $\{\mathcal{M}_0, \mathcal{M}_1\}$ of $\mathcal{Z}$, while heat records the dissipative channel accompanying driven irreversible change.

Writing the corresponding configuration-space density as $\rho(Z,t)$, the overdamped dynamics are equivalently

$$\partial_t \rho = -\partial_Z J, \qquad J = -\mu\, \rho\, \partial_Z U(Z;\lambda_t) - D\, \partial_Z \rho, \qquad \mu = \gamma^{-1}, \quad D = \mu k_B T. \tag{37}$$

The instantaneous equilibrium density selected by the current landscape is

$$\rho^{\text{eq}}_{\lambda_t}(Z) = \frac{e^{-\beta U(Z;\lambda_t)}}{\int_{\mathcal{Z}} e^{-\beta U(Z';\lambda_t)}\, dZ'}, \qquad \beta = (k_B T)^{-1}. \tag{38}$$

Coarse-graining this density over the protected basins yields the logical probabilities:

$$P_i(t) = \int_{\mathcal{M}_i(t)} \rho(Z,t)\, dZ = \Pr[\Pi_t = i]. \tag{39}$$

The continuous relative-free-energy functional, i.e. the nonequilibrium free-energy offset above equilibrium, whose two-state reduction gives the discrete KL ledger, is

$$\mathcal{F}_{\text{neq}}[\rho_t; \lambda_t] \equiv k_B T\, D_{\text{KL}}\Big(\rho_t \parallel \rho^{\text{eq}}_{\lambda_t}\Big). \tag{40}$$

To connect this continuous ledger back to Section 2, fix $\lambda_t = \lambda$ momentarily and define the entropy-like metriplectic potential $S_\lambda[\rho] \equiv -D_{\text{KL}}(\rho \parallel \rho^{\text{eq}}_\lambda)$. With this sign choice, the symmetric bracket raises S_λ toward its equilibrium maximum 0, equivalently driving D_{KL} monotonically downward. The Smoluchowski dissipator is then the concrete metric operator $M_\rho \phi = -\partial_Z(D\,\rho\,\partial_Z\phi)$, or equivalently the functional bracket $(A, B)_\rho = \int_{\mathcal{Z}} D\,\rho\,\partial_Z(\delta A/\delta\rho)\,\partial_Z(\delta B/\delta\rho)\, dZ$. With this choice, the purely metric flow is $\partial_t\rho|_{\text{met}} = M_\rho(\delta S_\lambda/\delta\rho) = -\partial_Z J$, reproducing Eq. (37), and

$$(S_\lambda, S_\lambda)_\rho = \int_{\mathcal{Z}} D\,\rho\,\left[\partial_Z \ln \frac{\rho}{\rho^{\text{eq}}_\lambda}\right]^2 dZ = \frac{\dot{\Sigma}_{\text{irr}}}{k_B},$$

so that $k_B T\, \dot{S}_\lambda = T\,\dot{\Sigma}_{\text{irr}}$. This is the continuous Smoluchowski realization of the abstract metriplectic relation $\dot{S} = (S, S) \geq 0$. The open-system bracket here does not impose $(H_{\text{sys}}, A) = 0$ on the bead potential $U(Z; \lambda)$. At fixed λ it dissipates the bead's nonequilibrium free energy relative to the thermal bath, while conservation of total energy belongs only to the enlarged bead-plus-reservoir description. Returning to the full time-dependent case, the exact metriplectic ledger for $\mathcal{F}_{\text{neq}}[\rho_t; \lambda_t]$ is

$$\frac{d}{dt}\mathcal{F}_{\text{neq}}[\rho_t; \lambda_t] = \dot{W}_{\text{ctrl}}(t) - T\,\dot{\Sigma}_{\text{irr}}(t), \qquad \dot{W}_{\text{ctrl}}(t) = \dot{\lambda}_t \int_{\mathcal{Z}} \rho_t(Z)\, \left.\frac{\partial U(Z;\lambda)}{\partial\lambda}\right|_{\lambda=\lambda_t} dZ, \tag{41}$$

with irreversible entropy-production rate

$$T\,\dot{\Sigma}_{\text{irr}}(t) = \int_{\mathcal{Z}} \frac{J_t(Z)^2}{\mu\,\rho_t(Z)}\, dZ \geq 0. \tag{42}$$

When the control is held fixed, $\dot{\lambda}_t = 0$, so the metric flow alone gives

$$\frac{d}{dt} D_{\text{KL}}(\rho_t \parallel \rho^{\text{eq}}_\lambda) = -\frac{\dot{\Sigma}_{\text{irr}}(t)}{k_B} \leq 0. \tag{43}$$

This is the physical-strain picture in continuous form: control moves the equilibrium target, while the dissipative metric flow relaxes the density along the negative functional gradient of D_{KL} and exchanges the corresponding heat through the bath. At any finite driving speed $\dot{\lambda}_t \neq 0$, the density cannot remain exactly on $\rho^{\text{eq}}_{\lambda_t}$; the control continuously displaces the target and the metric current continuously chases that lag, making finite-time transport strictly dissipative. Under fixed control, the monotone decay in Eq. (43) is the macroscopic Second Law in this coarse-grained setting, equivalently the H-theorem for the Smoluchowski dynamics.

4.1.3 Metastability and Reset Surgery

Near either minimum $Z_i(\lambda)$, the potential is locally

$$U(Z; \lambda) \approx U(Z_i(\lambda); \lambda) + \frac{1}{2}\kappa_i(\lambda)\big(Z - Z_i(\lambda)\big)^2, \qquad \kappa_i(\lambda) > 0, \tag{44}$$

so each basin is locally restoring. The curvature κ_i sets the intrawell relaxation time $\tau_{\text{relax},i} \sim \gamma/\kappa_i$, while the barrier height $\Delta U_i = U(Z^\ddagger;\lambda) - U(Z_i;\lambda)$ controls the Kramers escape time $\tau_{\text{escape},i}^{-1} \propto \exp[-\Delta U_i/(k_B T)]$. Condition I is therefore the regime $\tau_{\text{relax},i} \ll \tau_{\text{op}} \ll \tau_{\text{escape},i}$: the bead may wander within a basin, but basin membership remains stable over the operational window. Condition II says that the wells may be moved or reshaped slowly while preserving local restoring structure and basin disjointness, so the logical state Σ_t is transported without erasure. The ideal $C_{\text{ex}} \to 0$ limit must therefore be read as a formal asymptote or double-scaling statement: at fixed T and fixed ΔU_i, taking $\tau_{\text{op}} \to \infty$ would eventually violate metastability and erase the bit by ambient Kramers hopping.

Condition III is clearest when the device is reset into the deeper left basin. The control lowers the barrier, tilts the landscape left, and restores the double well after probability from the shallower right basin has been funneled into the left one. At the logical level, both input classes $\Sigma_{t_0} = 0$ and $\Sigma_{t_0} = 1$ map to the single output class $\Sigma_{t_f} = 0$. That is the many-to-one merger of world-tubes shown in Figure 1: after the protocol, the final basin no longer identifies the initial one.

If the bead is known with certainty to start in the left basin, then the same barrier manipulation can be arranged as a one-to-one transport of an already occupied tube while the right leg of the pair of pants remains empty; no macroscopic support is compressed and no logical erasure occurs. The irreversible case begins only when both legs carry probability mass and are forced into one trunk. Then the occupied coarse-grained support is genuinely contracted, and the missing retrodictability must be expelled through the metric sector.

The physical counter-operation is a Szilard-type expansion, not algebraic anti-diffusion. Suppose the bead is known to be in the left well after reset, but the observer wants to retrodict the two admissible predecessor branches. The time-reversed control protocol $\lambda^\dagger(t)$ lowers the central barrier without tilting into either well and expands the allowed macroscopic support from the left basin to both basins. The density then diffuses under the same positive Smoluchowski mobility until, in the symmetric limit, it occupies the two wells equally. To remain isothermal during this one-to-many expansion, the metric port imports thermal energy from the fluid bath, with the quasistatic symmetric intake $Q_{\text{in}} \approx k_B T \ln 2$ up to finite-time excess. The branch variable is thereby physically re-instantiated as expanded support rather than recovered by reversing the sign of time.

The control sector merely initiates this surgery by shifting the instantaneous equilibrium target $\rho^{\text{eq}}_{\lambda_t}$ into the left basin. Because the actual density $\rho_t(Z)$ is initially stranded on the right, a steep KL gradient appears. The metric sector—encoded by the symmetric bracket and, in the overdamped reduction, by the current $J_t(Z)$—then relaxes the state along the negative gradient of D_{KL}, equivalently up the gradient of $S_\lambda = -D_{\text{KL}}$, and thereby carries the physical port load. Equation (42) makes this explicit: the frictional cost $T\,\dot{\Sigma}_{\text{irr}}(t) = \int_{\mathcal{Z}} J_t(Z)^2/(\mu\,\rho_t(Z))\,dZ$ is what shrinks $D_{\text{KL}}(\rho_t \parallel \rho^{\text{eq}}_{\lambda_t})$, with heat exported during compression and imported during the controlled isothermal expansion just described. The bead's exact position is only a visualization aid; the information processing is defined by the driven dynamics of the protected partition on $\mathcal{Z}$.

4.1.4 The Accounting Clash

The same two-well model lets the abstract and physical ledgers be read side by side. Here P^{eq}_λ denotes the equilibrium basin-occupancy vector induced by the optical potential, whereas P is the actual coarse-grained state created by the control protocol. Shannon entropy measures uncertainty in P; Eq. (25) measures the retained logical-level displacement of P from P^{eq}_λ. Eqs. (41)–(43) give the exact density-level meaning of that distinction: control moves the equilibrium target $\rho^{\text{eq}}_{\lambda_t}$,

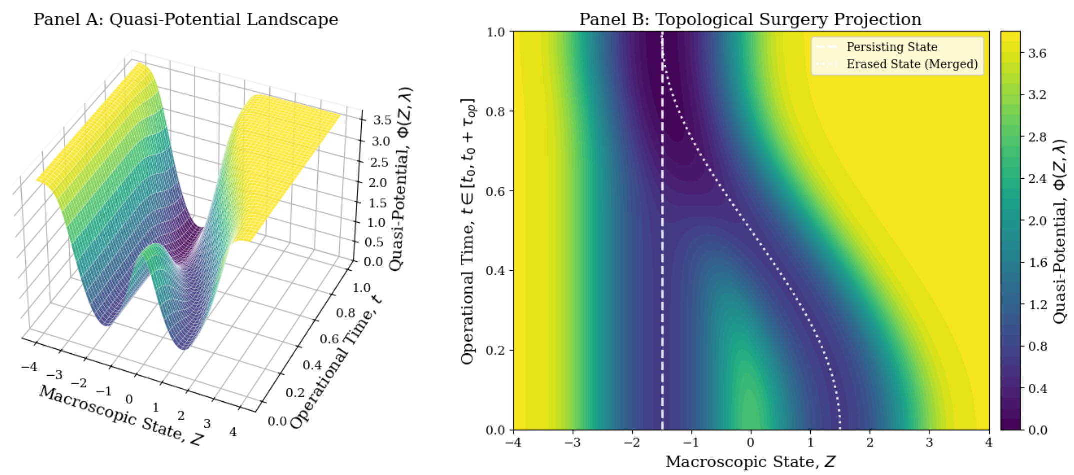

Figure 1: Pair-of-pants topology for colloidal-bit erasure in the extended mesoscopic space $\mathcal{E} = \mathcal{Z} \times \mathcal{T}$, with Z the coordinate on the one-dimensional readout space $\mathcal{Z}$. The two disjoint input world-tubes $\mathcal{W}_0$ and $\mathcal{W}_1$ persist during encoding, then a control protocol bends the right tube across the spatial axis and merges it into the left tube, yielding a single output tube and the many-to-one geometry of `RESET TO 0`.

and dissipation relaxes the full density ρ_t along the negative KL gradient. The vector P is the corresponding two-state reduction, exact when intrabasin structure is locally equilibrated. The textbook result is therefore a special symmetric case, not the general ledger.

In the perfectly symmetric benchmark,

$$P_{\text{sym}}^{\text{eq}} = \left(\frac{1}{2}, \frac{1}{2}\right). \tag{45}$$

If the bead is known to occupy the right well, so that $P = (0, 1)$, then

$$\mathcal{H}(P) = -\big[(0 \ln 0) + (1 \ln 1)\big] = 0, \qquad D_{\text{KL}}\Big(P \parallel P_{\text{sym}}^{\text{eq}}\Big) = \ln 2. \tag{46}$$

Thus the stored physical strain is $k_B T \ln 2$. For the logical reset of an unknown input ensemble $P_{\text{in}} = (1/2, 1/2) \to P_{\text{out}} = (1, 0)$, the coarse-grained Shannon drop is also $\Delta\mathcal{H}_{\text{cg}} = \ln 2$. Symmetry is therefore the special point at which abstract and physical nats happen to coincide numerically.

For the deliberately tilted landscape introduced above,

$$P_{\text{asym}}^{\text{eq}} = (0.999, 0.001), \qquad P = (0, 1). \tag{47}$$

The trapped pure state still has zero Shannon self-entropy,

$$\mathcal{H}(P) = 0, \tag{48}$$

so a purely abstract self-entropy calculation sees no stored information. The coarse-grained nat count instead gives

$$\mathcal{N} = D_{\text{KL}}\Big(P \parallel P_{\text{asym}}^{\text{eq}}\Big) = \left(0 \ln \frac{0}{0.999}\right) + \left(1 \ln \frac{1}{0.001}\right) = \ln(1000) \approx 6.91. \tag{49}$$

Here the vanishing term is understood in the standard information-theoretic sense $\lim_{x\to 0^+} x\ln x = 0$. Hence the corresponding coarse-grained nonequilibrium free-energy offset is

$$\mathcal{J}^{\mathrm{CCE}}_{T,\min} = k_B T \ln(1000) \approx 6.91\, k_B T. \tag{50}$$

Within the two-state reduction, this coincides with the coarse-grained free-energy bias of the wells,

$$\Delta F_{\mathrm{bias}} = -k_B T \ln \frac{p_1^{\mathrm{eq}}}{p_0^{\mathrm{eq}}} = k_B T \ln \frac{0.999}{0.001} \approx 6.91\, k_B T. \tag{51}$$

If the bead is then released or reset into the favored left well, the final state is $P_{\mathrm{out}} = (1, 0)$, whose stored nat count is

$$\mathcal{N}_{\mathrm{zero}} = D_{\mathrm{KL}}\Big(P_{\mathrm{out}} \,\|\, P^{\mathrm{eq}}_{\mathrm{asym}}\Big) = \ln\Big(\frac{1}{0.999}\Big) \approx 10^{-3}. \tag{52}$$

Under the same locally equilibrated two-state assumption, Eq. (28) therefore predicts that relaxing the prepared right-well state into the favored left well exports essentially the full $6.91\, k_B T$. The contrast with abstract bookkeeping is immediate. If the same reset is applied when the bead is already in the favored left well, the state is again perfectly known and its Shannon self-entropy is still $\mathcal{H}(P) = 0$. Abstractly, resetting a known 0 and a known 1 therefore look thermodynamically identical. Physically they are opposite: resetting the suspended right-well state releases $\approx 6.91\, k_B T$ through the metric port, whereas resetting the already relaxed left-well state releases essentially nothing beyond protocol overhead.

That is the accounting point. The Shannon quantity $\Delta\mathcal{H}_{\mathrm{cg}}$ remains the correct measure of logical distinguishability loss for a reset map, but it is not the full physical strain stored in an asymmetric landscape. At the exact density level the relevant ledger is $D_{\mathrm{KL}}(\rho \,\|\, \rho^{\mathrm{eq}})$; in the locally equilibrated two-state reduction used here this descends to $D_{\mathrm{KL}}(P \,\|\, P^{\mathrm{eq}})$. In the symmetric benchmark the two ledgers agree only accidentally; in the tilted landscape they separate immediately.

A similar separation may plausibly arise in actively stabilized digital hardware, but the present note does not derive a CMOS ledger from a transistor-level reservoir model. Establishing such a mapping would require a specified equilibrium reference state together with a careful separation among stored node energy, metastability barriers, leakage-maintenance power, and cycle-by-cycle switching dissipation. For that reason, no numerical CMOS nat count is claimed here. The conservative conclusion of the colloidal example is simply that asymmetric protected distinctions can store far more physical strain than the abstract one-bit Shannon ledger records, even when the logical distinguishability change is only $\ln 2$.

4.2 Cross-Substrate Bookkeeping Summary

Table 1 should be read at two levels. At the density level, the exact reference state is $\rho^{\mathrm{eq}}_{\boldsymbol{\lambda}}(Z) \propto \exp[-\sum_i \lambda_i X_i(Z)]$, where the multipliers λ_i are fixed by the chosen reservoir model. After coarse-graining into logical classes, the induced equilibrium weights take the form $P_i^{\mathrm{eq}} \propto \Omega_i \exp[-\sum_j \lambda_j X_j(i)]$ (or equivalently after absorbing Ω_i into effective class free energies). The nat metric therefore does not change across platforms, but its explicit form does depend on the level of description and on the reservoir convention used to define the multipliers.

One should also distinguish the two thermodynamic expenditures that Table 1 places side by side. The total generalized work required by a CCE operation over an operational window τ_{op} splits into

$$W_{\mathrm{total}} = \int_{t_0}^{t_0+\tau_{\mathrm{op}}} \dot{W}_{\mathrm{maint}}(t)\,\mathrm{d}t + W_{\mathrm{surgery}}. \tag{53}$$

Table 1: Illustrative conserved quantities X_i and Gibbs multipliers λ_i entering the density-level equilibrium reference state $\rho^{\mathrm{eq}}(Z) \propto \exp(-\sum_i \lambda_i X_i(Z))$. After coarse-graining into logical classes, the induced weights carry the same multipliers together with basin-degeneracy factors Ω_i. The final column lists representative foundational references for each encoding class.

Encoding class	Conserved quantity(ies) X_i	Reservoir parameter(s) fixing λ_i	Example	Refs.
Thermal encoding	Energy E	Bath temperature T fixing $\lambda_E = \beta = (k_B T)^{-1}$	Two-state thermal encoding in a dissipative bath	[13, 4]
Superconducting flux encoding	Magnetic flux Φ	Bias circuit fixing λ_Φ	Fluxoid states in a SQUID or superconducting loop	[15, 8]
Spin or rotational encoding	Angular momentum L	Spin/rotation reservoir fixing λ_L	Single-domain nanomagnet or spin-torque device	[5, 21]
Electrochemical encoding (multiplexed)	Charge Q and ionic occupation numbers N_i	Voltage/chemical-potential reservoir fixing λ_Q, λ_{N_i} (e.g. $\lambda_{N_i} = -\beta\mu_i$ in isothermal conventions)	Ion- and voltage-stabilized membrane states	[10, 1]
Flux-spin hybrid encoding (multiplexed)	Magnetic flux Φ and angular momentum L	Coupled reservoirs fixing λ_Φ, λ_L	Superconducting-spin hybrid circuits or magnetic Josephson structures	[14, 2]

The first term is maintenance: the ongoing resource flow required to preserve Condition I by holding a metastable barrier or nonequilibrium gradient against fluctuations. The second is surgery: the operational work required to deliberately enact Condition III, including any reversible free-energy change of the controlled landscape or state and, crucially, the metric-sector port load—written in the thermal case as $T\dot{\Sigma}_{\mathrm{irr}}$ with a sign fixed by the protocol convention. A reset exports load when protected classes are intentionally merged; a retrodictive expansion imports the conjugate load while support is reopened. In passively stable substrates, such as superconducting flux or structurally robust spin encodings, $\dot{W}_{\mathrm{maint}}$ may be negligible, so the visible bookkeeping is dominated by switching. In actively sustained substrates, such as optical traps or electrochemical gradients, external work must be supplied continuously just to keep the protected distinction intact. For electrochemical encodings, $\sum_i \Delta\mu_i \, \dot{N}_i(t)$ is the chemical power required to run ion pumps against leakage currents before any reset or expansion event adds its own port load. This distinction helps explain why biological computation is metabolically expensive: ionic and voltage gradients must be actively maintained even before a logical surgery event adds further exchange. In that active regime, the standing maintenance term is the dominant thermodynamic tax on keeping a CCE world-tube from dissolving into ambient noise. A resting neuron, for example, spends ATP continuously to hold ionic concentrations far from electrochemical equilibrium; that expenditure preserves the membrane's protected distinction before any explicit reset or spike-related merger occurs. The biological design pressure is therefore not merely to reduce the cost of isolated erasures, but to maximize how much useful reversible transport and conditional routing can be extracted per unit of standing maintenance burden. Put differently: once a substrate already pays heavily to keep its world-tubes intact, unnecessary irreversible mergers and expansions become especially costly.

5 Conclusion

This note proposes conservation-congruent encodings as a minimal physical definition of information at the macroscopic level. Information is not an abstract label floating free of matter, but a protected coarse-grained distinction specified by a substrate, a readout map π, a logical projection Π_t, and metastable world-tubes that remain operationally resolvable over a finite window. The metriplectic split supplies the dynamics: the Poisson sector stabilizes and transports the distinction, while the metric sector is the thermodynamic port through which many-to-one merger exports conserved load and controlled one-to-many expansion imports it.

The three defining conditions summarize the framework's operational content: metastable confinement for persistence, transverse restoring structure for quasistatic transport with vanishing excess expenditure only in the infinite-time limit, and irreversible tube merger together with controlled expansion for logical surgery. The exact physical nat count is the density-level relative entropy $D_{\mathrm{KL}}(\rho \parallel \rho^{\mathrm{eq}})$ to the appropriate equilibrium reference state. When the logical partition is an exact locally equilibrated lumping, this reduces to the coarse-grained ledger $D_{\mathrm{KL}}(P \parallel P^{\mathrm{eq}})$ on the logical classes. Substrate dependence enters not by changing that metric, but through the conserved quantities, basin degeneracies, and Gibbs multipliers that define the reference state. In the canonical thermal optical-tweezer model, $\rho_\lambda^{\mathrm{eq}}$ is the Boltzmann density of the landscape and P_λ^{eq} its induced basin-occupancy vector, so the familiar $k_B T \ln 2$ benchmark is the special symmetric limit of the coarse-grained ledger, not the generic rule for an asymmetric landscape. Shannon entropy accordingly survives inside the CCE picture as the abstract ledger of logical distinguishability, but not as the full thermodynamic accounting of stored strain, maintenance, and bidirectional channel exchange.

CCE therefore critiques abstract information theory not by discarding it, but by locating it within a broader conservation-grounded accounting of physical information. It isolates the minimal geometric and dynamical conditions under which a macroscopic distinction counts as a physically meaningful encoding, and shows how reversible transport, irreversible merger, and controlled expansion are grounded in conservation law. The maintenance/surgery split further shows why active substrates, especially biological ones, face a qualitatively different optimization problem from passively stable hardware: they must preserve protected distinctions continuously while rationing the additional port load of logical surgery.

References

[1] David Attwell and Simon B. Laughlin. An energy budget for signaling in the grey matter of the brain. *Journal of Cerebral Blood Flow & Metabolism*, 21(10):1133–1145, 2001.

[2] Burm Baek, William H. Rippard, Samuel P. Benz, Stephen E. Russek, and Paul D. Dresselhaus. Hybrid superconducting-magnetic memory device using competing order parameters. *Nature Communications*, 5:3888, 2014.

[3] Charles H. Bennett. Logical reversibility of computation. *IBM Journal of Research and Development*, 17(6):525–532, 1973.

[4] Antoine Bérut, Artak Arakelyan, Artyom Petrosyan, Sergio Ciliberto, Raoul Dillenschneider, and Eric Lutz. Experimental verification of landauer's principle linking information and thermodynamics. *Nature*, 483(7388):187–189, 2012.

[5] Jr. Brown, William F. Thermal fluctuations of a single-domain particle. *Physical Review*, 130(5):1677–1686, 1963.

[6] Massimiliano Esposito and Christian Van den Broeck. Second law and landauer principle far from equilibrium. *Europhysics Letters*, 95(4):40004, 2011.

[7] Peter David Fagan. Towards a physical theory of intelligence. Unpublished companion manuscript, 2026. Background companion manuscript for the broader CCE program; citation metadata provisional.

[8] Jonathan R. Friedman, Vijay Patel, W. Chen, S. K. Tolpygo, and J. E. Lukens. Quantum superposition of distinct macroscopic states. *Nature*, 406(6791):43–46, 2000.

[9] Miroslav Grmela and Hans Christian Öttinger. Dynamics and thermodynamics of complex fluids. I. development of a general formalism. *Physical Review E*, 56(6):6620–6632, 1997.

[10] A. L. Hodgkin and A. F. Huxley. A quantitative description of membrane current and its application to conduction and excitation in nerve. *The Journal of Physiology*, 117(4):500–544, 1952.

[11] Richard Jordan, David Kinderlehrer, and Felix Otto. The variational formulation of the fokker–planck equation. *SIAM Journal on Mathematical Analysis*, 29(1):1–17, 1998.

[12] Yonggun Jun, Momčilo Gavrilov, and John Bechhoefer. High-precision test of landauer's principle in a feedback trap. *Physical Review Letters*, 113(19):190601, 2014.

[13] Rolf Landauer. Irreversibility and heat generation in the computing process. *IBM Journal of Research and Development*, 5(3):183–191, 1961.

[14] Jacob Linder and Jason W. A. Robinson. Superconducting spintronics. *Nature Physics*, 11(4):307–315, 2015.

[15] J. E. Mooij, T. P. Orlando, L. Levitov, Lin Tian, Caspar H. van der Wal, and Seth Lloyd. Josephson persistent-current qubit. *Science*, 285(5430):1036–1039, 1999.

[16] Philip J. Morrison. Bracket formulation for irreversible classical fields. *Physics Letters A*, 100(8):423–427, 1984.

[17] Philip J. Morrison. A paradigm for joined hamiltonian and dissipative systems. *Physica D: Nonlinear Phenomena*, 18(1-3):410–419, 1986.

[18] Juan M. R. Parrondo, Jordan M. Horowitz, and Takahiro Sagawa. Thermodynamics of information. *Nature Physics*, 11(2):131–139, 2015.

[19] Udo Seifert. Stochastic thermodynamics, fluctuation theorems and molecular machines. *Reports on Progress in Physics*, 75(12):126001, 2012.

[20] David A. Sivak and Gavin E. Crooks. Thermodynamic metrics and optimal paths. *Physical Review Letters*, 108(19):190602, 2012.

[21] J. C. Slonczewski. Current-driven excitation of magnetic multilayers. *Journal of Magnetism and Magnetic Materials*, 159(1-2):L1–L7, 1996.

[22] Joanna A. Vaccaro and Stephen M. Barnett. Information erasure without an energy cost. *Proceedings of the Royal Society A*, 467(2130):1770–1778, 2011.