Epistemic Limits of the Embedded Observer

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Abstract

The Platonic observer is replaced by a physically embedded computational system. Bounded by the fundamental limits imposed by Conservation-Congruent Encoding (CCE), the observer is forced to project a massively large set of microscopic realities into a strictly smaller set of macroscopic equivalence classes. Consequently, state evolution under exact reversible dynamics becomes intractable within and between all embedded observers. Furthermore, a continuously modeled microscopic state space necessitates infinite encoding precision and unbounded operational resources to model exactly, rendering the continuum physically unobservable and forcing reliance on discrete macroscopic equivalences. These facts demonstrate that predicting/retrodicting the precise trajectory of the universe is unobtainable for embedded observers and demarks a fundamental epistemic constraint.

1 Introduction

Let an observer $\mathcal{O}$ utilize a physical encoding substrate $\mathcal{M}$ to process and store information about a microscopic physical system $\mathcal{E}$, which possesses state space $\Omega_{\mathcal{E}}$. Related physical limits on inference and universal computational capacity motivate this embedded-observer setting [1, 2]. The first two theorems treat the case in which $\Omega_{\mathcal{E}}$ is an effectively finite discrete state space; the final theorem treats the putative continuum as a stronger counterfactual limit.

2 Embedded Observer Information Axioms

Axiom 1 (Information Capacity Bounds)**.** *The observer's ability to store information is strictly bounded by its physical embedding:* $\mathcal{M} \subseteq \mathcal{O} \subset \mathcal{E}$.

By the Bekenstein-Hawking area law [3, 4], maximum encoding capacity is strictly monotone and finite for any bounded physical substrate: $I_{\max}(\mathcal{M}) \leq I_{\max}(\mathcal{O}) < I_{\max}(\mathcal{E}) < \infty$.

Axiom 2 (Information Dynamics Bounds)**.** *Encoded states are physical distinctions whose creation, preservation, and erasure must respect conserved quantities. In accordance with Landauer's principle and the Conservation-Congruent Encoding (CCE) framework [5, 6], irreversible information processing operations (e.g., erasure or merging) of* ΔI_{irr} *nats demand a proportional export of entropy into physical resource channels. For exchanged conserved quantities* Q_a *and their intensive conjugates* λ_a*, this CCE resource cost is strictly bounded:* $\sum_a \lambda_a \Delta Q_a \geq \Delta I_{\text{irr}}$.

3 Embedded Observer Epistemic Limit Theorems

Theorem 1 (Epistemic Limits in State)**.** *An isomorphic, complete microscopic model of* $\mathcal{E}$ *or* $\mathcal{O}$ *is physically forbidden. The observer must utilize an informational compression, mapping the finite microstate space* $\Omega_{\mathcal{E}}$ *into a strictly smaller finite set of discrete macroscopic encodings* Z.

Proof. An exact model of $\mathcal{E}$ requires an active encoding capacity $I(\mathcal{M}) \geq I_{\max}(\mathcal{E})$, violating the bounds of Axiom 1. Thus, the system is forced to use a many-to-one projection $\pi_\Phi : \Omega_{\mathcal{E}} \to Z$. Because $|Z| \ll |\Omega_{\mathcal{E}}|$ and the capacity of Z is finite, any metastable encoding basin $B_z = \pi_\Phi^{-1}(z) \subset \Omega_{\mathcal{E}}$ possesses a vast multiplicity of microstates. Therefore, there exist distinct microstates $x_1 \neq x_2 \in B_z$ that map to the exact same macroscopic encoding z, rendering them strictly indistinguishable to the observer. □

Definition 1 (Coarse-Grained Mixing)**.** A dynamical system with exact reversible map T exhibits coarse-grained mixing relative to the coarse-graining π_Φ if there exists an initial macrostate z_0 and a discrete time $\tau > 0$ such that the microscopic forward evolution of the basin $B_{z_0} = \pi_\Phi^{-1}(z_0)$ intersects multiple distinct macroscopic partitions: $|\{\pi_\Phi(T^\tau x) : x \in B_{z_0}\}| > 1$.

Theorem 2 (Epistemic Limits in Dynamics)**.** *The exact microscopic trajectory of* $\mathcal{E}$ *is unconditionally intractable within the observer's projection* π_Φ*. Furthermore, for systems exhibiting coarse-grained mixing, even the macroscopic future and past states cannot be uniquely predicted or retrodicted.*

Proof. By Theorem 1, the observer is restricted to a macroscopic initial state z_0 at discrete time t_0, which corresponds to a basin B_{z_0} containing multiple indistinguishable true microstates ($x_1, x_2 \in B_{z_0}$). Because the observer's formal model inherently loses the initial microscopic information, the exact deterministic trajectory realized by the universe is unconditionally intractable.

At the fundamental level, the true internal and external states evolve under an exact, deterministic, and reversible unit-step map T, so that $T_\tau = T^\tau$ for elapsed discrete time steps $\tau \in \mathbb{Z}_{>0}$. For any macrostate z, define the sets of distinct macroscopic futures and pasts by

$$\mathcal{F}_\tau(z) = \{\pi_\Phi(T_\tau x) : x \in \pi_\Phi^{-1}(z)\}, \qquad \mathcal{P}_\tau(z) = \{\pi_\Phi(T_\tau^{-1} x) : x \in \pi_\Phi^{-1}(z)\}.$$

For dynamics exhibiting coarse-grained mixing, localized microscopic differences are dispersed across the finite system. For some $\tau \in \mathbb{Z}_{>0}$, the forward evolutions of these initially indistinguishable microstates cross different macroscopic partition boundaries, yielding diverging macroscopic futures such that $|\mathcal{F}_\tau(z_0)| > 1$.

Conversely, macroscopic dissipation and retrodictive incompleteness emerge whenever $|\mathcal{P}_\tau(z_0)| > 1$, directly as an artifact of the many-to-one projection, which irreversibly merges distinct microscopic histories into shared macroscopic attractors.

The universe realizes exactly one deterministic microscopic trajectory. To operationally resolve this true trajectory from the macroscopic degeneracy, the observer must irreversibly process $I_{\mathrm{dec}}(\tau)$ additional nats of specification. By Axiom 2, this discrimination demands a physical resource payment $\sum_a \lambda_a \Delta Q_a^{\mathrm{dec}} \geq I_{\mathrm{dec}}(\tau)$. Because coarse-grained mixing continuously smears indistinguishable microstates across multiple macroscopic partitions, this informational deficit $I_{\mathrm{dec}}(\tau)$ strictly grows over time. Paying this cumulative energetic debt inevitably exhausts the finite conserved-resource capacity of $\mathcal{O}$, rendering exact prediction physically intractable. □

The preceding theorems establish epistemic limits within a discrete microstate space. The final theorem demonstrates that a continuous microscopic state space is strictly unobservable. The continuum demands exact mathematical distinctions that no embedded physical system can finitely encode, sustainably preserve, or operationally resolve.

Theorem 3 (Epistemic Limit of the Continuum). *If the microscopic state space $\Omega_{\mathcal{E}}$ is a continuous manifold, then an exact embedded model of a microscopic state or trajectory is physically forbidden by finite encoding capacity and finite operational resources. Consequently, continuity cannot be physically established by a finite embedded observer; only finite coarse-grained equivalence classes are observable.*

Proof. Assume, counterfactually, that $\Omega_{\mathcal{E}}$ is a continuous manifold and that the observer attempts an exact physical encoding. To isolate a single continuous coordinate q to a finite resolution ϵ over a bounded chart of size L, the observer's memory $\mathcal{M}$ would be forced to physically instantiate at least $\ln(L/\epsilon)$ nats of distinction. Exact specification would therefore demand an unphysical limit

$$I_{\text{exact}}(q) = \lim_{\epsilon \to 0} \ln(L/\epsilon) = \infty.$$

An exact embedded static model therefore demands $I(\mathcal{M}) = \infty$, which violently contradicts the geometric capacity bound $I_{\max}(\mathcal{M}) \leq I_{\max}(\mathcal{O}) < \infty$ established in Axiom 1.

Furthermore, resolving continuous dynamics catastrophically amplifies the epistemic failure established in Theorem 2. Because the observer is restricted to a macroscopic basin B_z containing uncountably many continuous microstates, discriminating the true state requires an unbounded refinement procedure. As the number of resolvable candidate cells approaches exact continuity ($N \to \infty$), the required informational specification diverges: $\Delta I_{\text{irr}} \to \infty$.

By Axiom 2, this irreversible specification demands a proportional physical resource payment:

$$\sum_a \lambda_a \Delta Q_a \geq \Delta I_{\text{irr}}.$$

As exact continuum discrimination sends $\Delta I_{\text{irr}} \to \infty$, the cumulative energetic debt diverges. The observer mathematically exhausts its finite conserved-resource budgets before exact discrimination can be completed. Therefore, an embedded observer is structurally cut off from the continuum. Any continuous model fitted to the observer's finite observational records $z \in Z$ remains physically underdetermined. The continuum may serve as a useful mathematical abstraction, but it cannot be physically proven from within a finite embedded observer. □

4 Conclusion

Epistemic incompleteness is not a philosophical interpretation; it is a fundamental physical constraint. For any embedded observer, the limits of finite encoding capacity dictate that the observable universe is necessarily coarse-grained and discrete. The same informational compression that makes a macroscopic model possible simultaneously destroys exact predictability. Because indistinguishable microstates exhibit coarse-grained mixing, exact reversible dynamics remain permanently intractable to the observer. Assuming a continuous microscopic state space does not circumvent this barrier; it fatally compounds it. The continuum makes an unphysical demand for infinite precision and unbounded operational resources, placing exact states beyond physical realization. Therefore, predicting the deterministic trajectory of the universe, or establishing physical continuity, remains out of reach for any finite embedded observer.

References

[1] David H. Wolpert. Physical limits of inference. *Physica D: Nonlinear Phenomena*, 237(9):1257–1281, 2008.

[2] Seth Lloyd. Computational capacity of the universe. *Physical Review Letters*, 88(23):237901, 2002.

[3] Jacob D. Bekenstein. Black holes and entropy. *Physical Review D*, 7(8):2333–2346, 1973.

[4] Stephen W. Hawking. Particle creation by black holes. *Communications in Mathematical Physics*, 43(3):199–220, 1975.

[5] Rolf Landauer. Irreversibility and heat generation in the computing process. *IBM Journal of Research and Development*, 5(3):183–191, 1961.

[6] Peter David Fagan. Conservation-Congruent Encoding. Preprint (v2), April 24, 2026.